Some Nuclear Chemistry Problems

1. Tritium (^{3}H) decays by beta emission with a half-life of 12.26 years. A sample of tritiated compound has an initial activity of 0.833 Bq. Calculate the number N_i of tritium nuclei in the sample initially, the decay constant k, and the activity after 2.50 years.

2. Over geological time, an atom of ^{238}U decays to a stable ^{206}Pb atoms by the emission of eight alpha emissions, each of which leads to the formation of one helium atom. A geochemist analyzes a rock and finds that it contains 9.0×10^{-5} mL of helium (at STP) per gram and 2.0×10^{-7} g of ^{238}U per gram. Estimate the age of the mineral, given that the half life of ^{238}U is 4.47×10^{9} years.

3. The half lives of ^{235}U and ^{238}U are 7.04×10^{8} and 4.47×10^{9} years respectively, and the present abundance ratio is $^{238}U/^{235}U = 137.7$. It is thought that their abundance ratio was 1 at some time **before** our earth and solar system were formed about 4.5×10^{9} years ago. Estimate how long ago the supernova occurred that supposedly produced all the uranium isotopes in equal abundance, including the two longest lived isotopes, ^{238}U and ^{235}U.

4. The beta decay of ^{40}K that is a natural part of the body makes all human beings slightly radioactive. An adult weighing 70.0 kg contains about 170 g of potassium. The relative abundance of ^{40}K is 0.0118%, its half-life is 1.28×10^{9} years, and its beta particles have an average kinetic energy of 0.55 MeV. (There are 1.602×10^{-13} J/MeV)

   a. Calculate the total activity of ^{40}K in this person.
   b. Determine (in rad per year) the annual radiation absorbed dose arising from this internal ^{40}K.

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1) $A = A_0 e^{-kt}$ and $A = -\frac{dN}{dt} = kN$

$A_0 = 0.833\ Bq \quad t_{1/2} = 12.26$ year

$k = \frac{\ln 2}{12.26 \text{ years}} = 5.6537 \times 10^{-2}\ yr^{-1}$

$= \frac{1\ yr}{(365 \times 24 \times 60 \times 60)\ s} \times 5.6537 \times 10^{-2} \frac{1}{yr}$

$= 1.7928 \times 10^{-9}\ s^{-1}$

$N_0 = \frac{A_0}{k} = \frac{0.833\ \frac{^3H}{s}}{1.7928 \times 10^{-9} \frac{1}{s}} = 4.6464 \times 10^{8}\ {}^3H$ nuclei

$A_{2.5\,yrs} = A_0 e^{-kt} = 0.833\ Bq\ e^{-5.6537 \times 10^{-2} yr^{-1} \times 2.5\ yr}$

$= 0.833\ e^{-0.1413}$

$= 0.833 \times 0.8682\ Bq$

$= 0.723\ Bq$

2) $^{238}U \rightarrow {}^{206}Pb + 8\,{}^{4}_{2}He + 6\,{}^{0}_{-1}\beta$

$t_{1/2} = 4.47 \times 10^{9}$ yr

$$\frac{N(^{238}U)}{N_0(^{238}U)} = \frac{N_0(^{238}U) - N(^{4}He)/8}{N_0(^{235}U)} = e^{-kt}$$

$$k = \frac{\ln 2}{t_{1/2}}$$

$$N_0(^{238}U) = N(^{238}U) + \frac{N(^{4}He)}{8}$$

$$N(^{235}U) = \frac{2.0 \times 10^{-7}\ g}{238\ g/mol} \times 6.022 \times 10^{23}\ \frac{atom}{mol}$$

$$= 5.06 \times 10^{14}\ \text{atoms}$$

$$N(^{4}He) = 9.0 \times 10^{-5}\ mL \times \frac{1\ L}{10^{3}\ mL} \times \frac{1\ mol}{22.4\ L} \times 6.022 \times 10^{23}\ \frac{atom}{mol}$$

$$= 2.42 \times 10^{15}\ \text{atoms}$$

$$N_0(^{235}U) = 5.06 \times 10^{14} + \frac{2.42}{8} \times 10^{15}$$

3.03×10^{14}

$$= 8.09 \times 10^{14}\ \text{atoms}$$

2) cont.

$$\frac{N(^{235}U)}{N_0(^{235}U)} = \frac{5.06 \times 10^{14} \text{ at}}{8.09 \times 10^{14} \text{ at}} = 0.625$$

$$0.625 = e^{-\left(\frac{\ln 2}{4.47 \times 10^9 \text{ yrs}}\right) t}$$

$$0.625 = e^{-1.551 \times 10^{-10} \text{ y}^{-1} t}$$

$$\ln 0.625 = -1.551 \times 10^{-10} \text{ y}^{-1} t$$

$$t = \frac{\ln 0.625}{-1.551 \times 10^{-10} \text{ y}^{-1}} = \left(\frac{-0.470}{-1.551 \times 10^{-10}}\right) \text{ yr}$$

$$= 3.0 \times 10^9 \text{ yrs}$$

3.0 billion years ago rock was formed.

3) con't.

$$\ln 137.7 = 8.295 \times 10^{-10} t$$

$$t = \frac{\ln 137.7}{8.295 \times 10^{-10}} \text{ yrs}$$

$$= \frac{4.9251}{8.295 \times 10^{-10}} \text{ yr}$$

$$= 5.9 \times 10^{9} \text{ years}$$

6 billion years ago

a nova created the uranium

3) $$\frac{N(^{238}U)}{N(^{235}U)} = 137.7$$

$$\frac{N_0(^{238}U)}{N_0(^{235}U)} = 1.0$$

① $$N(^{238}U) = N_0(^{238}U)\, e^{-\left(\frac{\ln 2}{4.47\times10^{9}}\right)t}$$

② $$N(^{235}U) = N_0(^{235}U)\, e^{-\left(\frac{\ln 2}{7.04\times10^{8}}\right)t}$$

Divide ① by ②

$$137.7 = 1.0\,\frac{e^{-\left(\frac{\ln 2}{4.47\times10^{9}\,yr}\right)t}}{e^{-\left(\frac{\ln 2}{7.04\times10^{8}\,yr}\right)t}}$$

$$= e^{\left(\frac{\ln 2}{7.04\times10^{8}} - \frac{\ln 2}{4.47\times10^{9}}\right)t}$$

$$= e^{\left(9.846\times10^{-10} - 1.551\times10^{-10}\right)t}$$

$$137.7 = e^{8.295\times10^{-10}\,t}$$

4) a) $A = kN \qquad k = \frac{\ln 2}{t_{1/2}}$

$k = 5.415 \times 10^{-10}\ yr^{-1}$

$= 1.717 \times 10^{-17}\ s^{-1}$

mass $^{40}K = 0.000118 \times 1.70\,g = 0.0201\,g$

$N(^{40}K) = 0.0201\,g \times \frac{1\ mol}{40.08} \times 6.022 \times 10^{23}\ \frac{atom}{mol}$

$= 3.03 \times 10^{20}$ atoms ^{40}K

$A = 1.717 \times 10^{-17}\ s^{-1} \times 3.03 \times 10^{20}$ atoms

$= 5.20 \times 10^{3}\ \frac{atoms}{s}$ or $\frac{disintegrations}{second}$

4 b) Energy released per year is

$$3.154 \times 10^{7}\,s \times 5.20 \times 10^{3}\,\frac{\beta}{s} \times 0.55\,\frac{MeV}{\beta} \times 1.602 \times 10^{-13}\,\frac{J}{MeV}$$

$$= 1.45 \times 10^{-2}\,J \quad \text{or } 1.45\,cJ$$

$$\frac{1.45\,cJ}{70.0\,kg} = 2.1 \times 10^{-2}\,\frac{cJ}{kg}$$

$$= 2.1 \times 10^{-2}\,rad$$

$$= 21\,m\,rad$$